

Solutions, Tutorial 3

1.

$$\mathbf{E} = E_x \hat{x}$$

$$\mathbf{i}_\perp = \sigma_{\parallel} E_{\parallel} = \sigma_{\parallel} E_z = \sigma_{\parallel} \cdot 0 = 0$$

$$\mathbf{B} = B_z \hat{z}$$

$$\mathbf{i}_\perp = \sigma_p \mathbf{E}_\perp + \sigma_H \frac{\mathbf{B} \times \mathbf{E}_\perp}{B} = \mathbf{i}_p + \mathbf{i}_H$$

$$\mathbf{i}_p = \sigma_p E_x \hat{x} = 0.8 \cdot 0.1 \hat{x} = 0.08 \hat{x}$$

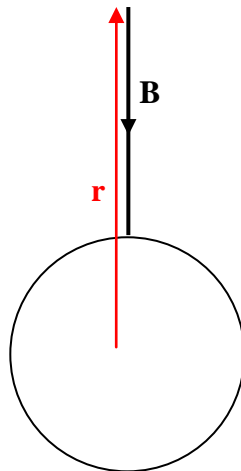
$$\mathbf{i}_H = \sigma_H \frac{(B_z \hat{z}) \times (E_x \hat{x})}{B_z} = 1.2 \cdot 0.1 \hat{y} = 0.12 \hat{y}$$

$$\tan \alpha = \frac{i_H}{i_p} = \frac{0.12}{0.08} \Rightarrow$$

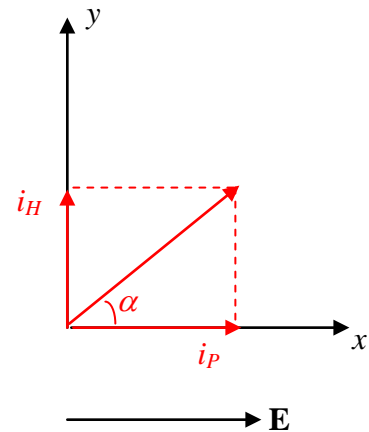
$$\alpha = 56^\circ$$

Note that we never needed σ_{\parallel} , since E_{\parallel} was zero.

2.



$$r = R_E + 1000 \text{ km}$$



The geomagnetic field strength is:

$$B = \sqrt{B_r^2 + B_\theta^2} = \sqrt{\left(B_p \frac{R_E^3}{r^3} \cos \theta\right)^2 + \left(\frac{B_p}{2} \frac{R_E^3}{r^3} \sin \theta\right)^2}$$

With $\theta = 0^\circ$ this reduces to

$$B = B_p \frac{R_E^3}{r^3} = B_p \frac{R_E^3}{(R_E + 1000 \text{ km})^3} = 0.646 B_p = 40.1 \mu\text{T},$$

since $B_p = 62 \mu\text{T}$ (Fälthammar p. 85)

The induced electric field is given by

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

Since all the angles are 90° , and we are only interested in the absolute value of E, we get

$$E = vB = vB_p \frac{R_E^3}{(R_E + 1000 \text{ km})^3}$$

Thus

$$E = 0.28 \text{ V/m}$$

3. a)

$$\mathbf{v} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

$$F = mg$$

$$\theta = \frac{\pi}{2}, r = R_E + 1000 \text{ km}, a = 8.0 \cdot 10^{22} \text{ Am}^2 \Rightarrow$$

$$B = \frac{\mu_0 a}{4\pi r^3} = 2.0 \cdot 10^{-5} \text{ T}$$

Right angles, so

$$v = \frac{mg}{qB} = \frac{m \cdot 7.3}{1.6 \cdot 10^{-19} \cdot 2.0 \cdot 10^{-5} \text{ T}} = 2.3 \cdot 10^{24} \text{ m}$$

∴

$$v_e = 2.3 \cdot 10^{24} \cdot 0.91 \cdot 10^{-30} = 2 \cdot 10^{-6} \text{ ms}^{-1}$$

$$v_o = 2.3 \cdot 10^{24} \cdot 16 \cdot 1.67 \cdot 10^{-27} = 0.06 \text{ ms}^{-1}$$

b)

$$v_{E \times B} = \frac{E}{B} = \frac{10 \cdot 10^{-3}}{2.0 \cdot 10^{-5} \text{ T}} = 500 \text{ ms}^{-1}$$

$$\frac{v_{E \times B}}{v_g} = \frac{500}{0.06} = 8140$$

4.

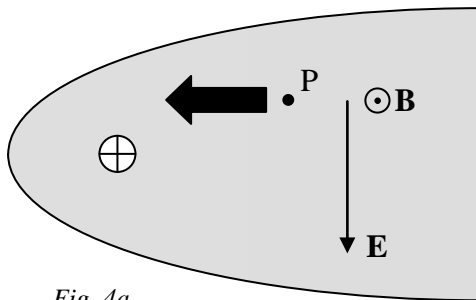


Fig. 4a

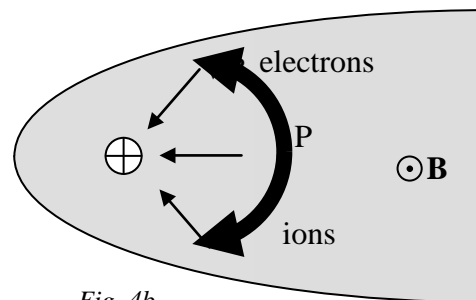


Fig. 4b

$$|\mathbf{v}| = \left| \frac{\mathbf{E} \times \mathbf{B}}{B^2} \right| = \frac{E}{B} = \frac{2.5 \cdot 10^{-3}}{120 \cdot 10^{-9}} = 20.8 \text{ km s}^{-1}$$

5.

Same as 1 (mistake by Tomas...)

6.

I choose $n_e = 1 \text{ cm}^{-3}$, and assume that the magnetosphere contains 50% protons and 50% oxygen ions. Then

$$\rho = (0.5 \cdot m_p + 0.5 \cdot m_O) n_e = (0.5 + 0.5 \cdot 16) m_p n_e = 0.5 \cdot 17 \cdot m_p n_e = 1.4 \cdot 10^{-20} \text{ kg/m}^3.$$

I approximate the magnetospheric volume by the volume of a cylinder with length $100 R_E$ and radius $10 R_E$:

$$V = \pi \cdot (10 R_E)^2 \cdot 100 R_E \approx 8 \cdot 10^{24} \text{ m}^3.$$

The mass of the magnetosphere then becomes

$$\rho V \approx 110 \text{ tonnes}$$

7.

Pressure balance between kinetic and magnetic pressure gives

$$\rho_{sw} v_{sw}^2 = \frac{B^2}{2\mu_0}$$

For a dipole field:

$$B^2 = B_r^2 + B_\theta^2 = \left(\frac{\mu_0 a}{2\pi r^3} \cos \theta\right)^2 + \left(\frac{\mu_0 a}{4\pi r^3} \sin \theta\right)^2$$

In the equatorial plane $\theta = 90^\circ$, and we get

$$B^2 = \left(\frac{\mu_0 a}{4\pi r^3}\right)^2$$

If we assume that the solar wind contains only protons

$$\rho = n_e m_p$$

and the pressure balance becomes

$$n_e m_p v^2 = \frac{\mu_0^2 a^2}{16\pi^2 r^6} \frac{1}{2\mu_0}$$

Solving for v , we get

$$v = \left(\frac{\mu_0 a^2}{32\pi^2 n_e m_p r^6} \right)^{\frac{1}{2}}$$

With the given numbers, we get

$v = 504$ km/s, which is not a totally unusual solar wind speed.